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Published in:
Proceedings of the 10th Conference on Engineering Mechanics

Publication date:
1994

Document Version
Publisher's PDF, also known as Version of record

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STRUCTURAL RELIABILITY THEORY
PAPER NO. 136

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Solution Methods for Structures with Random Properties Subject to Random Excitation

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ABSTRACT

This paper deals with the lower order statistical moments of the response of structures with random stiffness and random damping properties subject to random excitation. The arising stochastic differential equations (SDE) with random coefficients are solved by two methods, a second order perturbation approach and a Markovian method. The second order perturbation approach is grounded on the total probability theorem and can be compactly written. Moreover, the problem to be solved is independent of the dimension of the random variables involved. The Markovian approach suggests transforming the SDE with random coefficients with deterministic initial conditions to an equivalent nonlinear SDE with deterministic coefficients and random initial conditions. In both methods, the statistical moment equations are used. Hierarchy of statistical moments in the Markovian approach is closed by the cumulant neglect closure method applied at the fourth order level.

INTRODUCTION

The response of the structure will be stochastic in the cases uncertainties in initial conditions or external loads or parameters of the constitutive relations are modelled using random variables or random fields which are functions of time or space. In 1980's, the analysis of the stochastic response of stochastic structural systems recieved a lot of attention, consequently a new field, "Stochastic Finite Element Method (SFEM)" was coined to stochastic mechanics to analyze MDOF large scaled structural systems using discrete approximations. The developments in this field are reviewed by Vanmarcke et al. (1986), Benaroya and Rehak (1988), Ghanem and Spanos (1991), Brenner (1991), Der Kiureghian et al. (1991), Kleiber and Hien (1992).

For the full quantification of uncertainty using random variables and random fields, joint probability density function (pdf) should be assigned. Estimations of the joint pdf for the random models are based on experimental tests, observations and engineering judgement. It is very difficult, usually impossible, to quantify the uncertainty in terms of joint pdf. In practice, only the lower order statistical moments of the joint pdf can be estimated accurately. This study considers that the uncertainty can be quantified only for the first and second order statistics. It, then, calculates the first and second order statistical moments of the stochastic response. Due to page limitations, only linear multi-degree-of-freedom (MDOF) structural systems with random stiffness and damping properties subject to white noise excitation is studied. For extensions to nonlinear systems, non-white noise excitations and other derivations, see Köylüoğlu, Nielsen and Çakmak (1994), Köylüoğlu (1995).

The equations of motion of linear MDOF systems with random parameters subject to white noise excitation multiplied with intensity matrix $\mathbf{q}$ of dimension $p \times r$ are

$$\mathbf{m}\ddot{\mathbf{V}}(\mathbf{X},t)+\mathbf{C}(\mathbf{X})\dot{\mathbf{V}}(\mathbf{X},t)+\mathbf{K}(\mathbf{X})\mathbf{V}(\mathbf{X},t)=\mathbf{q}\mathbf{R}(t) \qquad \mathbf{V}(\mathbf{X},0)=\dot{\mathbf{V}}(\mathbf{X},0)=\mathbf{0} \tag{1}$$

where $\mathbf{m}$, $\mathbf{C}(\mathbf{X})$ and $\mathbf{K}(\mathbf{X})$ are mass, random damping and random stiffness matrices of dimension $p \times p$. $\{\mathbf{R}(t), t \in]-\infty,\infty[\}$ is a vector of dimension $r \times 1$ denoting zero-mean stationary white noise processes, $E\big[\mathbf{R}(t)\big] = \mathbf{0}$, with auto-covariance function $E\big[R_\alpha(t_1)R_\beta(t_2)\big] = \delta(t_2 - t_1)$, $\alpha, \beta = 1,\dots,r$. $\mathbf{X}^T = [X_1,\dots,X_d]$ are zero-mean random variables, $E[\mathbf{X}] = \mathbf{0}$, with specified covariances $E[X_iX_j] = \kappa_{X_iX_j}$. $X_1,\dots,X_d$ are all assumed to be stochastically independent of $\mathbf{R}(t)$. The displacement $\mathbf{V}(\mathbf{X},t)$ and velocity $\dot{\mathbf{V}}(\mathbf{X},t)$ response processes of the nodal points are random partly because of the functional dependency of the external random excitation process and partly due to the random random variables $\mathbf{X}$.

PERTURBATION METHOD

Consider the Taylor expansion of the random matrices and response quantities with respect to the random variables $X_1,\dots,X_d$ from the mean value system, e.g.

$$\mathbf{C}(\mathbf{X}) = \mathbf{c}_0 + \mathbf{c}_iX_i + \frac{1}{2}\mathbf{c}_{ij}X_iX_j + \cdots \tag{2}$$

$$V_m(\mathbf{X},t) \simeq V_m(\mathbf{0},t) + V_{m,x_i}(\mathbf{0},t)X_i + \frac{1}{2}V_{m,x_ix_j}(\mathbf{0},t)X_iX_j + \cdots \qquad m=1,2,\dots,p \tag{3}$$

where $\mathbf{c}_0 = \mathbf{C}(\mathbf{0})$, $\mathbf{c}_i = \frac{\partial}{\partial x_i}\mathbf{C}(\mathbf{0})$, etc. $\mathbf{V}(\mathbf{0},t)$ indicate the response processes on condition of $\mathbf{X} = \mathbf{0}$. $V_{m,x_i}(\mathbf{0},t) = \frac{\partial}{\partial x_i}V_m(\mathbf{0},t)$, etc. Further, summation convention has been applied for the dummy indices $i,j = 1,\dots,d$. Use of (2) and (3) and retaining terms up to second order in the random variables provides the following approximation for the unconditional covariance $\kappa_{V_mV_n}(t)$ of the response processes,

$$\kappa_{V_mV_n}(t) = E\big[V_m(\mathbf{X},t)V_n(\mathbf{X},t)\big] \simeq E\big[V_m(\mathbf{0},t)V_n(\mathbf{0},t)\big]+$$

$$\Big\{E\big[V_{m,x_i}(\mathbf{0},t)V_{n,x_j}(\mathbf{0},t)\big] + \frac{1}{2}E\big[V_m(\mathbf{0},t)V_{n,x_ix_j}(\mathbf{0},t)\big]+$$

$$\frac{1}{2}E\big[V_n(\mathbf{0},t)V_{m,x_ix_j}(\mathbf{0},t)\big]\Big\}\kappa_{X_iX_j} \tag{4}$$

In order to evaluate the expectations on the right sides, SDE must be formulated specifying the development of $\mathbf{V}(\mathbf{0},t)$, $\dot{\mathbf{V}}(\mathbf{0},t)$ and of the partial derivatives $\mathbf{V}_{,x_i}(\mathbf{0},t)$, $\dot{\mathbf{V}}_{,x_j}(\mathbf{0},t)$, $\mathbf{V}_{,x_ix_j}(\mathbf{0},t)$, $\dot{\mathbf{V}}_{,x_ix_j}(\mathbf{0},t)$. These are obtained from partial differentiation of (1) with respect the random variables, evaluated at the mean structure $\mathbf{X} = \mathbf{0}$. All of these equations can next be cast into the following closed system of first order SDE with state vector $\mathbf{Z}(t)$. Note that $\mathbf{Z}(0) = \mathbf{0}$.

$$\dot{\mathbf{Z}}(t) = \mathbf{a}(\mathbf{Z},t)+\mathbf{b}\mathbf{R}(t)\ ,\ \mathbf{Z}(t) = \begin{bmatrix} \mathbf{V}(\mathbf{0},t) \\ \dot{\mathbf{V}}(\mathbf{0},t) \\ \mathbf{V}_{,x_i}(\mathbf{0},t) \\ \dot{\mathbf{V}}_{,x_i}(\mathbf{0},t)hfill \\ \mathbf{V}_{,x_j}(\mathbf{0},t) \\ \dot{\mathbf{V}}_{,x_j}(\mathbf{0},t) \\ \mathbf{V}_{,x_ix_j}(\mathbf{0},t) \\ \dot{\mathbf{V}}_{,x_ix_j}(\mathbf{0},t) \end{bmatrix}\ ,\ \mathbf{a}(\mathbf{Z},t) = \mathbf{A}\begin{bmatrix} \mathbf{V}(\mathbf{0},t) \\ \dot{\mathbf{V}}(\mathbf{0},t) \\ \mathbf{V}_{,x_i}(\mathbf{0},t) \\ \dot{\mathbf{V}}_{,x_i}(\mathbf{0},t) \\ \mathbf{V}_{,x_j}(\mathbf{0},t) \\ \dot{\mathbf{V}}_{,x_j}(\mathbf{0},t) \\ \mathbf{V}_{,x_ix_j}(\mathbf{0},t) \\ \dot{\mathbf{V}}_{,x_ix_j}(\mathbf{0},t) \end{bmatrix}\ ,\ \mathbf{b} = \begin{bmatrix} \mathbf{0} \\ \mathbf{m}_0^{-1}\mathbf{q}_0 \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \tag{5}$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\mathbf{m}_0^{-1}\mathbf{k}_0 & -\mathbf{m}_0^{-1}\mathbf{c}_0 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\mathbf{m}_0^{-1}\mathbf{k}_i & -\mathbf{m}_0^{-1}\mathbf{c}_i & -\mathbf{m}_0^{-1}\mathbf{k}_0 & -\mathbf{m}_0^{-1}\mathbf{c}_0 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ -\mathbf{m}_0^{-1}\mathbf{k}_j & -\mathbf{m}_0^{-1}\mathbf{c}_j & \mathbf{0} & \mathbf{0} & -\mathbf{m}_0^{-1}\mathbf{k}_0 & -\mathbf{m}_0^{-1}\mathbf{c}_0 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} \\ -\mathbf{m}_0^{-1}\mathbf{k}_{ij} & -\mathbf{m}_0^{-1}\mathbf{c}_{ij} & -\mathbf{m}_0^{-1}\mathbf{k}_j & -\mathbf{m}_0^{-1}\mathbf{c}_j & -\mathbf{m}_0^{-1}\mathbf{k}_i & -\mathbf{m}_0^{-1}\mathbf{c}_i & -\mathbf{m}_0^{-1}\mathbf{k}_0 & -\mathbf{m}_0^{-1}\mathbf{c}_0 \end{bmatrix} \tag{6}$$

The state vector $\mathbf{Z}(t)$ would have a dimension $N = 2p+4(pd)+2(pd^2)$ if it is constructed using all the random variables of the system. Then, the method would, certainly, suffer from the high dimension N and becomes not practiable, if p and, especially, d are large. Indeed, only state vectors of smaller dimension $N = 8p$ are needed to be considered where the state vector for certain i and j is used to calculate all the corresponding coefficients to $\kappa_{X_iX_j}$ in equation 4, one by one. There will be $\frac{d(d+1)}{2}$ many smaller systems to consider. (5) is a linear SDE with deterministic coefficients. $E[\mathbf{Z}(t)] = \mathbf{0}$ and covariances are obtained from moment equations.

MARKOVIAN METHOD

Since the random variables of the structural system has been assumed to be time-invariant, the following SDE represent the random variables with probability 1.

$$\dot{\mathbf{S}}(t) = \mathbf{0} \quad , \quad \mathbf{S}(0) = \mathbf{X} \tag{7}$$

where $\mathbf{S}(t)$ is a time-invariant dummy random process and the initial conditions $\mathbf{S}(0)$ are random with the same probabilistic structure as the random variables $\mathbf{X}$. (1) and (7) can next be combined into a closed system of equivalent 1st order SDE as in (5), but, the state vector $\mathbf{Z}(t)$ is now made up of the displacement and velocity vector and the random variables of the structural system.

$$\mathbf{Z}(t) = \begin{bmatrix} \mathbf{V}(t) \\ \dot{\mathbf{V}}(t) \\ \mathbf{S}(t) \end{bmatrix} \quad , \quad \mathbf{Z}_0 = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{X} \end{bmatrix} \quad , \quad \mathbf{a}(\mathbf{Z}, t) = \begin{bmatrix} \dot{\mathbf{V}}(t) \\ -\mathbf{m}^{-1}\mathbf{C}(\mathbf{S})\dot{\mathbf{V}}(t) - \mathbf{m}^{-1}\mathbf{K}(\mathbf{S})\mathbf{V}(t) \\ \mathbf{0} \end{bmatrix} \quad ,$$

$$\mathbf{b} = \begin{bmatrix} \mathbf{0} \\ \mathbf{m}^{-1}\mathbf{q} \\ \mathbf{0} \end{bmatrix} \tag{8}$$

Due to $\mathbf{R}(t)$ and as $\mathbf{Z}_0$ is independent of $\mathbf{R}(t)$, the state vector $\mathbf{Z}(t)$ is Markovian. The new state vector $\mathbf{Z}(t)$ will have a dimension $N = 2p + d$. Hence, the dimension of the state vector is proportional to the number of random variables. For very large d, this could be a drawback compared to the perturbation method.

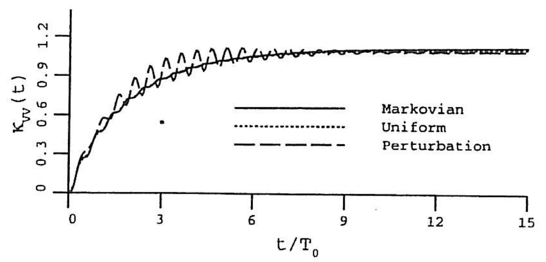
Since the drift vector is nonlinear in $\mathbf{Z}$, the statistical moment equations for this state vector are ordinary nonlinear differential equations where hierarchy of moments appear. In this study, the hierarchy is closed by the cumulant neglect closure method applied at the fourth order statistical moment level.

NUMERICAL EXAMPLE

A numerical example is worked out for a SDOF oscillator. The random parameters C and K are assumed to be mutually stochastically independent with the following following mean values ($E[\cdot]$) and variational coefficients ($v[\cdot]$). $m = 1.0, E[C] = c_0 = 0.1, v_C = 0.3, E[K] = k_0 = 1.0, v_K = 0.3, q = \sqrt{2c_0k_0}$. The corresponding variances of the displacement and velocity of the mean linear oscillator are both equal to 1. $\kappa_{VV}(t)$ obtained by the presented methods are compared to the so-called exact ones in Figures 1 and 2. Given $f_{\mathbf{X}}(\mathbf{x})$, the exact unconditional nonstationary variances $\kappa_{VV}(t)$ can be obtained by the application of the total probability theorem on the conditional nonstationary variances $\kappa_{VV}(\mathbf{X} = \mathbf{x})$.

$$\kappa_{VV}(t) = \int_{\mathbf{x}} \kappa_{VV}(\mathbf{X} = \mathbf{x}, t) f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \tag{9}$$

$f_{\mathbf{X}}(\mathbf{x})$ is assigned as uniform and triangular shaped distributions for C and K.



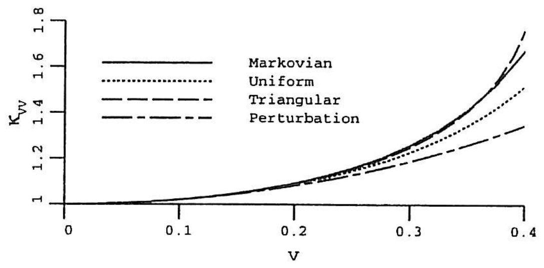


Figure 1) $\kappa_{VV}(t)$ versus $\frac{t}{T_0}$. Only K is random. Figure 2) Stationary κ_{VV} versus $v = v_C = v_K$.

CONCLUSION

The SDE with random coefficients resulting in dynamic SFEM problems are attacked by two methods, a second order perturbation approach and a Markovian method. Perturbation method is grounded on total probability theorem, yields a formulation independent of the dimension of the random variables involved, possesses divergent secular terms under the governing control of damping in the nonstationary regime and may handle coefficient of variations up to 25-30 percent. The Markovian approach results in hierarchy in the statistical moment equations. Closure by the cumulant neglect closure method applied at the fourth order level seems to be promising to attack problems with larger variability.

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